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A Survey of Changing of Units of Measure in First Grade through Eighth Grade Mathematics Materials

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A SURVEY OF CHANGING OF UNITS OF MEASURE IN FIRST GRADE THROUGH EIGHTH GRADE MATHEMATICS MATERIALS

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CHAPTER I

INTRODUCTION

Differences of opinions among mathematics and science teachers of ninth grade students exist regarding the ability of their students to change from one unit of measure to another unit of measure. The author contends, as a result of teaching eighth grade mathematics for four years, that students in the eighth grade do not have an adequate understanding of changing of units of measure in the English as well as in a fictional system. Therefore, a test was given to students to see if one of the two systems, fictional or non-fictional, was more difficult than the other. Difficulty in the students' performances in either system would indicate a need to find out how changing of units of measure is presently being taught.

I. THE PROBLEM

It was the purpose of this study (1) to compare the ability of eighth grade students to change units of measure in the English system and in a fictional system of measurement; (2) to survey four mathematics textbook series, grades one through six, and four series, grades seven and eight, on changing of units of measure; and (3) to survey three college level textbooks, which could possibly be utilized

in an arithmetic methods course, to see how changing of units of measure is suggested to be taught to children.

II. IMPORTANCE OF THE STUDY

The fascinating record of man's invention of the idea of numbers and the numerals used to represent these ideas has its earliest advances buried in pre-recorded history. As man's method of subsistence changed from hunting and fishing to more agrarian pursuits, his need for numbers undoubtedly changed from just simple counting of discrete objects to include measurement. The first recorded use of measure dates back to the first dynasty of Egypt, approximately 3400 B. C. More advanced forms of measure such as tables of weight, length, and area dating from 220 B. C. have been found on Babylonian clay tablets (5:635). The expansion of civilization and the increased use of measures helped to create the need to be able to change from one unit of measure to an equivalent unit of measure.

Measuring situations are some of the most common applications of number. However, the verbal communication of the use of number in measurement differs from profession to profession. For example, many mathematicians will not accept inches, feet, or gallons as being a part of the number and would ask for a statement such as "The length of the desk in inches is thirty-four." The scientist, on

the other hand, multiplies three feet by four pounds and arrives at twelve foot pounds. Swain states:

> The matter of writing units into equations is controversial. Engineers and physical scientists do not hesitate to write units in whenever this seems desirable. Mathematicians studiously avoid putting units in, though insisting upon full descriptions, with units specified, of all physical quantities that are symbolized in the work. A reason for this "purism" is that the theories underlying mathematical operations are framed in terms of numerical variables, and would be unduly and unnecessarily complicated in terms of measurement variables . . . The college mathematics professor usually requires his students to test the units as an aspect of the problem apart from the numerical or algebraic manipulation (7:202-3).

An outgrowth of these diverse approaches is a difference of opinion regarding the place of measurement in the elementary school mathematics curriculum. One faction would contend that, from the point of view of mathematics as a structural and logical system, there is no place for measurement in the curriculum. The other group might then argue that, from the point of view of children's needs, there is a very important place for measurement in their daily existence (4:174).

Among those who think that measurement knowledge is important is Charles H. D'Augustine. He feels that measurement is such a broad topic that the most that teachers can hope to do at the elementary level is to provide the children with certain basic concepts and principles relating to measurement that will be adaptable

to all types of measures they might encounter in the future (2:320). D'Augustine states:

> It is important to develop general conversion techniques for the more common measures that will be encountered in everyday life. However, it is equally important that children view the measure system in its broader perspective so that in the future they will be able to interpret and translate from one standard to another, working with measures uncreated as of now (2:318).

Through man's experiences involving conversion techniques within several units of the same measure, he should form two generalizations. First, in changing the expression of a measured quantity from larger to smaller units of measure, one multiplies the number of larger units by the number of smaller units contained in one of the larger units. Second, in changing the expression of a measured quantity from smaller units to larger units, a person divides the number of smaller units by the number of smaller units contained in one of the larger units (1:172).

As an expansion on D'Augustine's theory that children need to understand measurement, Westcott and Smith state that after some exposure to and experience with existing systems of weights and measure, children should be called upon to create new systems of weights and measures. In addition, children should have the opportunity actually to use their new system of weights and measures in some problem-solving situation. Therefore, since all weights and measures are

an idea (the products of man's mind), new ideas or modifications of existing ones are in order (3:154).

D'Augustine, Wescott, and Smith all maintain that an important part of teaching measurement is having students apply and work with measurement systems other than those commonly used; that is, the student should have the experience of working with a fictional system. However, since most students are consumers, they need to understand present-day measuring systems.

Today's advertising requires the consumer to be discerning in his knowledge of measurement. For example, a teacher posed this question to a class: "Bill had to decide which of two types of soda pop to buy, a half quart for 35¢ or a pint for 34 cents. If he likes both types of soda pop equally well, which would be the best buy?" This problem of comprehending packaging and measurement is also a problem for adults. Sometimes the housewife finds the unit cost (e.g., cost per pound, cost per ounce) of the giant size box of soap is higher than that of the regular size. Another advantage of knowing units of measure is that the uses of units of measure in science, industry, and medicine are often concerned with changing from one unit of measure to another unit of measure.

A search for experimental data relative to changing of units of measure was not beneficial. Although sources

which the author was able to find dealing with measurement were investigated, no research concerned with changing of units of measure was found.

III. LIMITATIONS OF STUDY

This survey does not take into consideration the methods which a classroom teacher could use that are not part of the textbook the teacher studied in college or the student text being used. Individual classroom teacher techniques to teach the changing from one unit of measure to another unit of measure, other than those presented in student textbooks and mathematics methods textbooks, were not investigated.

The study is further limited by a lack of research available for comparison of results on the topic of changing of units of measure. The test devised by the author did not involve the use of concrete objects but was entirely a paper and pencil test.

IV. DEFINITIONS OF TERMS USED

Throughout the entire survey of both the series and the methods texts, upon which this study is based, the terminology of the individual authors of these texts is used. Where there is a chance of misunderstanding, the terms are defined as they are surveyed. The term "changing

of units of measure" means to write an equivalent form that has the same value as the original form. The word "change" is left undefined as is necessary in the study of a logical system. The terms "converting," "transforming," "writing an equivalent name," and "renaming" all have the same meaning as "changing of units of measure."

There is a distinction between "measurement" and "changing of units of measure" made by the author of the thesis. Measurement as a topic includes concepts of area, linear measurement, volume, and weight. The particular system (English, metric, or fictional) is of less importance. When a student has been introduced to a system of measurement, it is then appropriate to study how to change from one unit of measure to another unit of measure in that system. However, although it is also quite possible to study various systems of measurement and never be concerned with changing from one unit of measure to another, this survey is concerned with changing from one unit of measure to another, and this survey is also concerned with the concept of changing from one unit of measure to another unit of measure in a system of measurement.

V. ORGANIZATION OF REMAINDER OF THE THESIS REPORT

Considering the desirability of a student's being able to change from one unit of measure to another unit of

measure in any system of measurement, the author presents in the following chapters the procedures followed in the construction of the test, the textbook series survey, and both of the texts for elementary children and for college students in training to teach arithmetic. The final chapter summarizes this material and presents recommendations based on the findings.

CHAPTER II

PROCEDURE

The topics contained in this chapter are the following: a discussion of the sample of the students used; the construction and classification of the test items; the results of the test responses; and the procedures used in the survey.

I. TEST ON CONVERTING UNITS

The sample used. A short test dealing with an understanding of changing of units of measure was given to one hundred forty-four eighth grade students on Mercer Island, Washington, by the author of this thesis. Only those eighth grade students who were present on a normal school day at North Mercer Junior High School became subjects for the study.

Each student had been given the Lorge-Thorndike group intelligence test in September, 1967. The results are generalized below:

Number of pupils	144
Range of scores	80 - 145
Mean	117.1
Median	122.0
Standard Deviation	11.1

The test on changing of units of measure was administered late in May, 1969, after the students had completed all the work on changing of units of measure. The test was administered at this time because ninth grade students are generally expected to be able to change one unit of measure to another. The text being used by these eighth graders was Exploring Modern Mathematics, Book II (Grade 8) published by Holt, Rinehart, Winston in 1963. Included in this are approximately two pages of review material on conversion of one unit of measure to another. The teacher's manual recommends review of the four pages in Book I (Exploring Modern Mathematics, Grade 7) of the same series.

Since these students had been exposed to changing of units of measure, the author of the thesis felt that they should have little or no difficulty with the sets of problems in the English or a fictional system. Therefore, a test was devised to determine if there were a difficulty. If there were any significant statistical differences in the eighth grade students' understanding of changing of units of measure in these two systems, then it would seem to point to a difficulty in the students' understanding of changing from one unit of measure to another unit of measure.

The test. The test was constructed by the author of the thesis and involved problems in changing from one unit

of measure in the English system to another unit of measure in the English system. A conversion table of the necessary English units was provided. Additional problems involving a change of units of measure in a fictional system were used. A conversion table of the fictional system was given to the students.

The construction of the fictional system was parallel to the English system. The English units of measure were replaced by a fictional name in each corresponding instance. The ratios of units were also changed. The eight fictional problems were arrived at by replacing the English units with their respective fictional units and varying the numerical values to provide whole number answers for the conversions.

A copy of the test that was used is given in the appendix. Table I shows the classification of the test items, whether they were E (English) or F (fictional). It also tells the number of correct responses that were given for each item.

The test consisted of eight problems involving changing of units of measure in the English system and eight problems involving the fictional system. Just as the two systems were constructed to be parallel, the problems were also parallel. A knowledge of the basic operations with whole numbers was needed to do the calculations. Taking the eight test items classified as English and the eight

TABLE I

CLASSIFICATION OF TEST ITEMS AND FREQUENCY DISTRIBUTION OF CORRECT RESPONSES

Item number	Classification	Correct responses
1.	E	54
2.	F	54
3.	F	24
4.	E	26
5.	E	25
6.	F	17
7.	F	86
8.	F	55
9.	F	38
10.	E	17
11.	E	61
12.	E	26
13.	E	50
14.	F	11
15.	F	91
16.	E	5

E = English

F = fictional

items classified as fictional, the author of the thesis placed each problem on a separate sheet of paper. The papers were then placed in a box. Then, the papers were drawn out of the box. Each problem, regardless of whether it was English or fictional, was numbered for the test in the order in which it came out of the box.

Results. A frequency distribution of the combined English and fictional test scores made by the one hundred forty-four eighth grade pupils of Mercer Island, Washington, are found in Table II. A total of 1,152 test responses on English problems was obtained. Two hundred fifty-four of these items were missed. Of the 1,152 test responses on the fictional problems, three hundred sixty-nine were incorrectly answered. These results in percentages are 22.6% of the English items were missed. Of the fictional items 31.5% were wrong. A frequency distribution of the pupils' raw scores for the English conversions and fictional conversions is found in Table III on page fifteen.

T test. To determine if the difference in the means of the English conversion subtest scores and the fictional system scores were statistically significant and not due to sampling error, a "t" test was used. A probability value for acceptance of the difference of the means as statistically significant was arbitrarily set by the author at "p" is less

TABLE II

FREQUENCY DISTRIBUTION OF COMBINED ENGLISH AND FICTIONAL TEST SCORES

Pupil scores	Frequency of pupil scores
16	12
15	14
14	21
13	16
12	21
11	21
10	10
9	6
8	4
7	8
6	6
5	2
4	2
3	0
2	0
1	1
Number of pupils	144
Mean	11.68
Median	12.68
Standard Deviation . . .	3.2712

TABLE III

FREQUENCY DISTRIBUTION FOR ENGLISH AND FICTIONAL SUBTESTS

English Conversions		Fictional Conversions	
Pupil score	Frequency of score	Pupil score	Frequency of score
8	35	8	20
7	42	7	26
6	25	6	30
5	23	5	27
4	9	4	19
3	4	3	10
2	5	2	10
1	1	1	0
		0	2
Mean of scores . . .	6.23	Mean of scores . . .	5.43
Median of scores . .	6.61	Median of scores . .	5.63
Standard Deviation .	1.6115	Standard Deviation .	1.6944

than .05. A "t" score of 6.79 was calculated. This "t" score indicates that the difference in the means would be expected in fewer than one out of one hundred cases as a function of sampling error. Therefore, the difference in the means of the English and fictional subtests is statistically significant. Formulas used in determining "t" are found in the appendix.

II. PROCEDURES USED IN THE SURVEY OF TEXTBOOKS

For the purpose of this survey, the following recent mathematics textbook series were chosen for grades one through six. In the survey and the report of the findings, these books are coded as Series A, Series B, Series C, and Series D. The appropriate grade level is noted when necessary.

A. Elementary School Mathematics
Second Edition, Addison-Wesley
Publishing Company, 1968.

B. Greater Cleveland Mathematics Program,
Science Research Associates, Incorporated, 1968.

C. Modern School Mathematics - Structure and Use
Houghton Mifflin Company, 1967.

D. Sets and Numbers, The L. W. Singer Company,
Incorporated, 1968.

The following mathematics textbook series were chosen for grades seven and eight. In the survey and reports of the findings, these books are coded as Series E, Series F,

Series G, and Series H. The appropriate grade level is noted when necessary.

E. Exploring Arithmetic, Webster Publishing Company, 1962.

F. Exploring Modern Mathematics, Holt, Rinehart, and Winston, 1968.

G. Mathematics, Structure and Skills, Science Research Associates, Incorporated, 1968.

H. School Mathematics, Addison-Wesley, 1967.

These textbooks were chosen because none of them had been published prior to 1962. These books should reflect, as much as possible, the present day mathematics programs. These textbooks series were also considered for adoption by the Mercer Island School District for the 1969-1970 school year.

To get a clear picture of the approach used in the textbook series being surveyed, the following questions were investigated.

(1) How and when is the changing of units of measure introduced?

(2) How is the changing of units of measure reinforced or further developed in later grade levels?

To answer these questions, the material presented as the course of study and the suggestions to teach the concepts in the course of study were analyzed. Notes were taken on the material in the textbooks as documented in the

indices. After the entire series had been surveyed, a report of the findings was written about each series. These reports comprise Chapter Three.

The following contemporary books, which could serve as texts in a methods course for elementary teachers of mathematics, were chosen for the survey. In the reports these books are coded as Text X, Text Y, Text Z.

X. Herbert F. Spitzer, Teaching Elementary School Mathematics. Houghton Mifflin Company, 1967.

Y. C. Alan Riedesel, Guiding Discovery in Elementary School Mathematics. Appleton-Century-Crofts, 1967.

Z. Howard F. Fehr and Josephine McKeeley Phillips, Teaching Modern Mathematics in the Elementary School. Addison-Wesley Publishing Company, 1967.

These methods textbooks were chosen because they were published after 1966 and thereby meet the criterion of being currently in use. The names of the writers were familiar to the author, and the textbooks were readily accessible for the use in the survey.

The question investigated for the methods textbooks is the following: How is the changing of units of measure presented? Notes were taken on items pertinent to this question. These findings are summarized in Chapter Four.

CHAPTER III

SURVEY OF THE TEXTBOOKS SERIES

A survey was made of four contemporary elementary textbook series, coded as Series A, Series B, Series C, and Series D in this chapter. Four contemporary seventh and eighth grade series coded as Series E, Series F, Series G, and Series H were also used. The teacher's manuals were used since they contained the instructions to the teacher as well as to the pupil. These series were surveyed to find out how the changing of one unit of measure to another was introduced, reinforced, and further developed.

I. SERIES A

First grade. Liquid measurement is introduced. The idea of matching sets is used to show that two pints are the same as one quart and that four cups are the same as one quart. The purpose of the lesson is for children to become exposed to a measurement that differs from linear measurement. Children are to complete matching exercises by experimenting with water and containers of different measurement sizes.

Second grade. Students are provided practice in linear measurement. They are to measure certain lengths in the English units and then in the metric units. Liquid

measurement is extended and enriched. The students are to experiment further with containers and water in order to work the exercises in the textbook.

Third grade. The teaching of liquid measurement is expanded further. The opportunity is given to the students to convert from one unit of measure to another by using containers and water or sand. Students are urged to verify results and settle arguments by actual demonstrations.

Fourth grade. A lesson is provided to practice changing from one unit of measure to another. The author of the textbook suggests that the teacher work through the discussion exercises with the children. The teacher should help the student verbalize the changes made when measures are combined and when the differences between measures are found. Also, the authors of the textbook advise that the teacher use a model on the chalkboard to help students visualize the problem. The children are encouraged to aid the teacher in using the units in order to express a given measurement. According to the textbook author, the children should understand that when they combine measures they sometimes get a number of smaller units which they can convert into one or more of the larger units. Further, the children should not be expected to remember all the rates of conversions, but they are expected to show

some skill in combining measures and finding differences between two given measures. The students should be allowed to handle semi-abstract representations as they make conversions. Additional work is provided in conversions with liquid measures and linear measures.

Fifth grade. A lesson is devoted to review of linear measurement and metric measurement. Students are given the chance of expressing measurements in more than one unit. One example is given: 1 ft. 14 in. = 2 ft. 2 in. The teacher is to give the children several examples of ways to express a given measurement with the greatest possible number of the larger units. The purpose of the lesson is to help students to formulate the general concept that the longer units give a smaller number for the length of an object. Supplementary exercises are provided.

Additional experience in the metric system is provided, and converting from one unit to another unit is expected. Students are to use a diagram of a meter stick given in the textbook. The teacher is requested by the authors of the textbook to provide several examples (of the meter stick) before assigning students any problems. The authors say that the teacher will have to work through many of the exercises with the children in order for them to understand conversions.

Volume concepts are reviewed. Conversions of measures of volume are made; however, only two conversion exercises are included. The authors recommend that the teacher solve the exercises with the students. The teacher is to spend a considerable amount of time demonstrating the use of three-dimensional models.

Sixth grade. There is a review on units of measure in the student text and in the supplementary exercises. Converting units of measure is involved in these exercises. The student is to add and to change each answer to the greatest number of units, for example:

$$\begin{array}{r} 5 \text{ gal. } 2 \text{ qt.} \\ +\ 3 \text{ gal. } 3 \text{ qt.} \\ \hline 8 \text{ gal. } 5 \text{ qt. or } 9 \text{ gal. } 1 \text{ qt.} \end{array}$$

Nine gallons, one quart is the correct answer.

II. SERIES B

First grade. This book introduces the concept of measuring the value of money and develops an understanding of the relationship between the penny and the nickel. A thorough understanding of the concept of measurement of value will come only with prolonged exposure, through a great deal of experience with handling coins in a meaningful situation, according to the authors of the textbook series.

This lesson emphasizes helping children recognize different sets of coins that have the same value. Students learn how many nickels are needed to make ten cents and what coins are needed to make thirteen cents (using coins largest in value).

Second grade. In introducing the concept of liquid measurement the authors provide no written work for the children because the authors of the textbook feel that the most profitable way to study liquid measure is to measure liquid. Conversion from one unit to another is not discussed.

Third grade. Standard units of liquid measure are introduced. The student explores the relative size of a cup, pint, quart, and gallon and learns to express the same amount of liquid with different units. The student uses his knowledge of multiplication to convert from one unit to another. Concepts are developed by using containers and water or by using diagrams.

Fourth grade. Measurement of length, areas, weights, and amounts of liquid are compared to standard units. Comparison is used to develop an understanding of numbers as a measure of quantity.

Fifth grade. The concept of measurement of an area is introduced. Measurement concepts are developed around the question, "How much?"

Sixth grade. Measurement of volume is introduced. The basic purpose of this development is to help the pupils to see the reasons that the study of volume is important and to give them continued practice in finding volume. The author of the textbook states in the teacher's notes that conversions of units need not be mastered at this time.

The metric system of measurement is investigated. The basic unit of measure for length in the metric system is the meter. Other metric units of linear measure are obtained either by successive division by ten or by successive multiplication by ten. Practice is provided in converting meters, decimeters, and centimeters. Metric units of weight are developed by use of physical models.

III. SERIES C

First grade. Liquid measure is introduced by using the concept of one-to-many matching as a special case of one-to-one matching. This is applied to the relationship of cups and pints in liquid measurement. The textbook authors stress that "one pint makes two cups," rather than "one pint = two cups." The equal sign is reserved for number sentences which give two names for the same number.

One pint is equivalent to two cups since they both measure the same amount of liquid. But, the units of measure are not identical things, so the equality is not used. The teaching procedure endeavors to provide the students with the idea of the one-to-two relationship by drawing two number lines on the board indicating that for each one-unit increase on the quart number line, there is a two-unit increase on the pint number line.

Second grade. This text introduces and develops the concept that one foot is equivalent to twelve inches by the direct comparison of a line segment containing twelve inches with a line segment of one foot. As many opportunities as possible are given to the student to make measurements involving feet and inches. Students are to be told by the teacher to construct a chart showing their heights, the lengths of their arms, feet, legs, and so forth. The textbook authors do not want lengths measured in feet and then in inches but want the answers in feet plus inches.

The concept of cups, pints, and quarts as units of liquid measure is reinforced. The purpose is to develop the student's understanding of the relationship between these units. The textbook authors advise that, because children find volume a difficult concept, the teacher must teach the relationship carefully. Conversion of liquid

measure is reviewed, and half-gallons and gallons are introduced. These authors state that for children to gain a really sound understanding of measurement and the relationship between units, children need many opportunities to experiment with physical objects. The textbook authors avoid statements such as "two cups equal one pint" because they feel that only equations should involve numbers. It is better to say "two cups are equivalent to one pint."

The concept of measuring the value of money is introduced. A comparison of units of money by the one-to-many matching is expanded. A dollar matches a set of one hundred pennies, one-to-one-hundred.

Third grade. One page of review of liquid measures is provided. Students are told to study the given chart and to complete the exercises. The given chart diagrammatically shows the relationship between cups, pints, quarts, and gallons. The idea of one-to-many matching is reinforced. Students should know that, for example, one quart may be practiced by using the one-to-many matching idea. Conversion is accomplished by addition.

The terms foot and yard are introduced as units of measure. The equivalency between feet and inches, and yards and feet is given. Methods of conversion are by matching. The following example is given. If one foot matches twelve

inches, then two feet match "a" inches, a = twenty-four. The student's book states that working with feet and inches necessitates the ability to multiply by twelve.

Fourth grade. The metric system is introduced. A comparison is made between it and the English system. No conversions are made from one metric unit to another. Conversions from the metric system to the English system are given and vice versa. Students must use multiplication to do the given conversions.

Fifth grade. One page of review of units of measures previously introduced is given. The authors of the textbook emphasize avoiding the use of one foot "equals" twelve inches. Equality means that two things are the same, and such is not the case here. It is correct to say that "one foot is equivalent to twelve inches."

Addition and subtraction of measures are given reinforcement by one page of exercises. First, students are to rename units of measure. Then, they are told when to add or when to subtract in the exercises.

The metric system is extended. Conversion from one unit of measure to another unit of measure in the metric system is presented. The fact that the metric system uses base ten is stressed. All conversions in the exercises given involve multiplication by a power of ten. Only approximations involving decimal answers are expected.

Sixth grade. The English and the metric systems of measurement are reinforced. The textbook authors mention in the English system there is no basic number for converting from one unit of measure to another. They also state that one must memorize the relationship between units. The review of the English system is directed toward practice in converting from one unit of measure to another. Students are to be helped in this process by having available a meter stick, a quart measure, a liter measure, a kilogram weight, and a pound weight in order to compare the corresponding units. The teacher is directed to work some examples that demonstrate converting from a smaller unit of measure to a larger unit.

The idea of function and rate is applied to converting units of measure to another unit of measure. All of the exercises in this lesson are of the type: given "m" find f (m). For example, the function equation for the rate three feet per one yard is $3 \times m = f(m)$. The activities are of the type: given the number of yards, m, find the number of feet, f (m). One could also use the equation to find m, given $f(m) = 15$. One could find the number of yards by solving the equation $3 \times m = 15$.

IV. SERIES D

First grade. The author of this thesis was not able to find exercises on changing one unit to another unit of measure.

Second grade. Liquid measurement is introduced at this level. The students are to make conversions involving cups, pints, and quarts by using containers and water. Linear measurement is presented; conversions involving feet and inches is presented by using a model of a foot, marked off in inches. Conversion involving quarters, half-dollars, dimes, and nickels is provided by using pictures of real money and/or play money.

Third grade. Weight measurement is presented in this grade. An example, 1 lb. 6 oz. = ? oz., is given. The students are given the answer of twenty-two. The reason stated is that one is to add sixteen and six because one pound measures sixteen ounces.

Measures of liquid are introduced by having the teacher demonstrate to the class the completion of a few of the problems and assigning the rest to be completed independently.

Linear measurement is extended. Conversions involving inches, feet, and yards are given. Pupils are introduced to this material by having the teacher demonstrate that when three 12-inch rulers are placed by a yardstick, the rulers and yardstick are the same length. The students are then to determine that one yard is also equal to thirty-six inches by substituting 12 inches for one foot. The teacher is to tell the students to memorize the equivalent

measure for inches, feet, and yards. Students are to work the exercises on this page by substituting equivalent measures in terms of inches. The following are examples.

3 yards 1 foot 7 inches = ? inches

36 inches + 12 inches + 7 inches = 55 inches

There are three pages devoted to this method.

Review practice is given in converting measurement about time. The authors of the textbooks state that they want the pupils to memorize the equivalent relationships between the measures pertaining to any of the characteristics which are commonly measured.

Fourth grade. Time measurement is reviewed and extended. The exercises provided may all be worked by using repeated addition or multiplication. The author of the textbook recommends that all conversion tables be memorized. The teacher is to explain what "convert into new units" means. Then the exercises are to be done independently by the students.

Money measurement exercises are provided. To find the total value of a collection of coins of given values, the denominations of the coins are added. With this method it is necessary to give the value of each coin in terms of cents before adding.

Measures of weights and their relationship are reviewed. Children use various kinds of scales to help in

making conversions. Practice in converting pounds to ounces and ounces to pounds is reviewed and extended. Physical models are used to help students in conversions. Liquid measurement is examined again. Children are given several examples by the teacher which require converting a given measurement to an equal measurement using a different unit.

Linear measurement is reviewed, and practice is provided in converting yards to miles and feet to miles. A review of converting of all types of units of measure is provided.

Fifth grade. Linear measurement is reviewed and further developed to include rods. Exercises are phrased so that the student looks for the number that makes a true equation such as the following: 2 rods = ? yards. The use of the equal sign when used with a unit of measure means equal in measure. A student can say that twelve inches equals one foot, but he must remember that he means the measures are equivalent, not the numbers.

Temperature measurement using the centigrade scale is introduced. Students make conversions from Fahrenheit to centigrade temperatures by reference to a side-by-side drawing of a centigrade and of a Fahrenheit scale of the same length.

Liquid measurement is studied again and extended to include the tablespoon and the fluid ounce. The teacher

is to provide several examples. Conversion exercises are written in the form of equations.

Sixth grade. Not only is linear measurement of English measures reviewed, but the concept of the square inch and of the acre is added. Conversion exercises are included.

Cubic measurement is also introduced. The author mentions that when finding the area or the volume, one must be sure that all the dimensions are in the same unit.

The method for converting from centigrade to Fahrenheit is first to multiply the centigrade reading by 1.8, then to add thirty-two degrees. Several conversion exercises are then given for the students to do.

In the review of measures of weight, the hundredweight is presented. Conversions are made by referring to a conversion table and substituting equivalent units of measure for given units.

The linear measures of the metric system are introduced. The student text states that the metric system has different units which are multiples of ten, so it is easy to convert from one unit to another. The following example is given.

$$\begin{aligned} 1\ \text{m} &= 10\ \text{dm} \\ &= 100\ \text{cm} \\ &= 1000\ \text{mm} \end{aligned}$$

The student must be able to answer questions of the following type before the teacher assigns any exercises.

(1) How do you change five meters to decimeters? Desired response: Since there are ten decimeters in one meter, in five meters there are (5 X 10) decimeters.

(2) How do you change two decimeters to centimeters? Desired response: Since there are ten centimeters in one decimeter, in two decimeters there are (2 X 10) centimeters.

V. SERIES E

Seventh grade. Area, volume, linear measurement, and the metric system are reviewed. Conversion tables are provided, and exercises involving changing from one unit to another unit are included.

Eighth grade. Area, volume, linear measurement, and the metric system are reviewed. Conversion tables are provided. Exercises involve changing from one unit of measure to another.

VI. SERIES F

Seventh grade. One chapter is devoted to measurement and error. Much of the material will already be familiar to the students, but new material is included.

The metric system is introduced. The teacher is to stress the ease in changing of units in this system.

Changing units involves only a repositioning of a decimal point, if decimal numerals are used. The changing of units in the metric system is demonstrated as being easier than in the English system.

Exercises are in the form of equations which the student is to complete to make a true sentence. Dry, liquid, and weight measures are discussed. Conversion exercises are provided. The textbook authors state that if the students are wondering at this point how the metric and the English units compare, a full study of this should be delayed until a development of the method of changing of units is presented later on in the textbook. The idea is further developed that unit symbols and numerals may be handled in the same way as fractional numerals.

Students are told that when one changes units, one either multiplies or divides by some number. Unit symbols are also changed. But, so far, the students do not have a very good way to check whether or not they are right. Maybe they multiplied when they should have divided. Perhaps the newly assigned unit symbol is wrong.

Students are led by the exploratory exercises to discover the idea that certain symbols might act like a multiplicative identity, using patterns learned with fractional numerals. As an example, the following shows the change of seventeen minutes into seconds.

$$(17 \text{ min.}) \frac{(60 \text{ sec.})}{(1 \text{ min.})} = 17.60 \text{ sec.} \frac{\text{min.}}{\text{min.}}$$

$$= 1020 \text{ sec.}$$

The above use of the multiplicative identity can be used twice or as many times as necessary. The authors of the textbook indicate that the above method should not be allowed to become a crutch for the students. They should be encouraged to do without it when they can and to use it when they must. Application of the use of the multiplicative identity method in the metric and other units of measure is presented.

Eighth grade. Conversion of different types of measures is reviewed, and the authors stress that the multiplicative identity concept should be reviewed by the teacher. This conversion method is used when applicable to all conversions of one and two dimensional units discussed in this book.

It is stated by the textbook authors that the method of handling unit symbols is not presented as a mathematical system; however, it would be possible to do so.

VII. SERIES G

Seventh grade. Liquid measurements and linear measurements are reviewed. Exercises involving conversions

are provided for both the English and the metric system. Exercises are in the form of equations for the student to complete to make a true statement.

Eighth grade. Students are asked to find the height of objects in inches and in centimeters by measuring with the appropriate instruments. Volume measurement is discussed. Conversions are given. Many diagrams and physical models are used by the students.

VIII. SERIES H

Seventh grade. Optional problems dealing with conversion in the metric system are supplied. Students are given a conversion table. Then they are asked to find the number that makes the given equation true.

A discussion of the general use of ratios as well as their use in converting from one unit of measure to another is presented. Students are told to study the following method for converting.

30 mi. per hr. to 44 ft. per sec.

$$\frac{30 \text{ mi.}}{1 \text{ hr.}} = \frac{30 \text{ mi.}}{60 \text{ min.}} = \frac{1 \text{ mi.}}{2 \text{ min.}} = \frac{5280 \text{ ft.}}{2 \text{ min.}} = \frac{2640 \text{ ft.}}{60 \text{ sec.}} = \frac{44 \text{ ft.}}{1 \text{ sec.}}$$

The emphasis of this lesson is on the general ideas involved in converting from one unit to another so that conversion of units can be applied in a wide variety of situations.

Eighth grade. Conversion between metric system and English system is provided because it involves working with decimal equivalences. The exercises present a variety of problems involving different units in the metric system.

IX. SUMMARY

The following summaries constitute what the author of the thesis found in each textbook series.

Summary of Series A. Conversion of units of measures is developed mainly through concrete experiences by the student. Students are helped to generalize about specific cases, with a different generalization for each case. The authors do not provide the student with any experiences with a fictional system of measurement.

Summary of Series B. Conversion of units is developed through concrete experiences of the student. Very little emphasis is on conversion of units of measure. Several conversion methods for specific cases are given. There are no fictional systems proposed to the students.

Summary of Series C. Many concrete experiences are provided for students. Conversions are made on one-to-many matching basis, or a rule for particular conversions is given. The idea of function is included for specific

conversions problems. No fictional system is introduced to the students.

Summary of Series D. Conversions are introduced by use of concrete experiences. Substituting equivalent units of measure for others obtained from conversion tables which have been memorized is taught in equations. Students learn that two different meanings may be attached to the equal sign and that the meaning is determined by its usage. Rules are also given for conversions. There are many pages devoted to converting measurement.

Summary of Series E. The students are given the conversion tables and examples. Exercises are provided. Students learn to make conversions by example.

Summary of Series F. The concept of using the multiplicative identity and fractional numerals is used by students to convert from one unit of measure to another.

Summary of Series G. Converting from one unit of measure to another is permitted by using physical models. There is very little conversion of units of measure using paper and pencil only.

Summary of Series H. The use of ratios in converting from one unit of measure to another is developed.

A summary of all textbooks and their presentation of changing of units of measure is given in Chapter V. Also included are the recommendations of the author of this thesis for the teaching of changing of units of measure and further studies that should be made.

CHAPTER IV

SURVEY OF ARITHMETIC METHODS TEXTS

A survey was made of three contemporary texts, which could be used in a course on methods of teaching elementary mathematics. These texts will be coded as Text X, Text Y, and Text Z in this chapter. These texts were surveyed to see what methods were recommended in the presentation of changing from one unit of measure to another unit of measure.

I. TEXT X

In the suggested sequence of steps in teaching measurement, the author of the textbook suggests that the teacher should "apply the theory" that the student be given the opportunity to calculate with measures and transform from one unit of measure to another. Emphasis is put on the fact that there are several names for the same length. An example is given on changing a length to inches.

2 ft. 7 in. (2 x 12 + 7) in. = 31 in.

This is called converting into inches.

In reference to the metric system the author of the textbook states that children will learn a "rule of thumb" by performing transformation in changing from the metric system to the English system.

II. TEXT Y

Riedesel discusses in one chapter two types of measurement, discrete and continuous. Discrete variables are such things as cost of postage stamps, size of families, and census enumeration. Continuous variables are such things as height, weight, time, temperature, and blood pressure. Specific reference is made to linear measurement, area, angles, volume, time, and calendar.

In this chapter Riedesel states that the English system of measure has a consistent base. Therefore, renaming is done in terms of the relationship between a measurement unit and the next unit that is greater or less in value. This is the only time that changing from one unit of measure to another is mentioned. There are no methods or examples given to explain how to change from one unit of measure to another unit of measure. The use of a fictional measurement system is not mentioned.

III. TEXT Z

Spitzer devotes one full chapter to measurement. He develops the teaching procedures which he feels are designed to develop understanding of measurement. He is concerned in this chapter with the concepts of linear measurement, weight measurement, the metric system, volume and capacity, angle measurement, and time. There is no

mention in any of these areas of how to teach a child to change from one unit of measure to another unit of measure. There is no suggestion of using a fictional system in the development of concepts of measurement.

IV. SUMMARY

The main concern of these three methods textbooks for teachers is upon the development of the student concept of how much a unit of measure is. The relationship between units is limited somewhat to the comparison of size. Use of concrete examples is stressed in all three texts. Two of the three books mention conversion or renaming units of measure. There is no mention of using a fictional system made up by the teacher and/or the student.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

It was the purpose of this study (1) to compare the ability of eighth grade students to change units of measure in an English system and in a fictional system of measurement; (2) to survey four different mathematics textbook series for grades one through six and four series for grades seven and eight on how the changing of units of measure is introduced and developed; and (3) to survey three college level textbooks, which could be used in an arithmetic methods course, to see how changing of units of measure is suggested to be taught to children. This final chapter presents a summary of the findings in each of these three areas. General conclusions, recommendations for teaching the changing of units of measure, and recommendations for further studies are set forth at the end of this chapter.

I. SUMMARY

Changing of units test. A test was given to eighth grade students involving problems in changing from one unit of measure to another. A test for determining the significance of the difference of the means on the English and the fictional systems was made.

Whether or not the results of the student test indicate any transfer of learning from the English to the fictional system is debatable because the amount of transfer is conditioned by many factors, among which are: age; mental ability; time interval between learning and transfer; "knowledge of directions"; favorable attitude toward the learning situation and efficient use of past experiences; method of study; and suitable organization of subject matter (8:205).

Textbook series. In the series surveyed there are differences in the terminology used to denote changing of units of measure. "Changing," "converting," "transforming," "writing an equivalent name," and "renaming" are terms which mean the same thing.

There is a difference in the number of exercises included in the textbooks which deal with changing from one unit of measure to another. One series had as many as thirty exercises at each grade level, while another had only ten exercises at each grade level. Some of the series taught students how to change from one unit of measure to another in one lesson, while other series spread out the lesson by incorporating it with related material.

In all elementary series the first concepts of changing of units of measure are introduced by the use of physical objects. Conversions are on a very simple basis.

A distinction between number and unit of measure as being a number is not always evident. The use of the equal sign with units of measure varies from series to series.

Introducing the concept of changing of units of measure with liquid toward the end of the first grade or the beginning of the second grade seems to be standard. The types of units of measures introduced in the textbooks series are not standard. The use of a fictional system of measurement was not mentioned.

Methods texts. In the three methods texts surveyed, only Text X and Text Y devoted a discussion to changing of units of measure. The other text did not discuss this topic. There was no mention of using a fictional system of measurement by any of the authors.

II. CONCLUSIONS

(1) There is a significant difference in the ability of eighth grade pupils to convert from one unit of measure to another in a fictional and an English system.

(2) The terminology used varies from one mathematics textbook series to another.

(3) There is no general method for changing from one unit of measure to another as found in textbook series or methods texts.

(4) Experience with fictional systems of measures was not provided in any of the series.

(5) Teachers will not always find detailed methods for teaching conversions in either school textbooks or methods textbooks.

III. RECOMMENDATIONS FOR TEACHING CHANGING OF UNITS OF MEASURE

(1) Changing of units of measure should be introduced through children's use of models and other concrete experiences. Hull states that, "A combination of abstract presentations and concrete examples yields a distinctly greater functional efficiency than either method alone" (8:93). Clark and Fehr stress the importance of sensory experience and proceeding from the concrete to abstract in concept building (8:339). The initial phase, therefore, should be comprised of concrete experience in changing from one unit of measure to another.

(2) Provisions for maintenance of skill in changing from one unit of measure to another unit of measure should be provided. Exercises involving conversions of units of measure, once the concept has been established, should be provided periodically during each school year. This will help strengthen and develop the student's facility to change from one unit of measure to another. The opportunity should

also be provided for the students to engage in many different types of conversion exercises.

(3) Experiences in fictional measurement systems made up by the student and/or the teacher should be provided. Following the recommendation of D'Augustine, Wescott, and Smith this would help the student extend his concept of changing of units of measure. The student would then be prepared for any new measurement system that might confront him in the future.

(4) The identity method of changing from one unit of measure to another unit of measure should be used more extensively at the seventh and eighth grade level. The use of this method strengthens and gives the student practice in applying the multiplicative identity property, the associative and commutative properties of real numbers as applied to units of measure.

IV. RECOMMENDATIONS FOR FURTHER STUDIES

The following recommendations by the author of this thesis are a result of this survey. These recommendations would require research beyond the limits of the undertaking reported.

(1) A study should be made to determine if the teaching of a fictional measurement system has an effect on the transfer of learning to other fictional systems or

to the English system. A study of this nature would help to answer questions regarding the desirability of teaching students a fictional system in order to improve their understanding of the conversion of common units of measure.

(2) A study should be made to see if there is a difference between the ability of students to do conversion problems using only paper and pencil and their ability to do conversion problems using paper, pencil, and concrete objects. The use of concrete materials in changing from one unit of measure to another is stressed by textbooks in the development of the concept. All students may not have made the transition from the concrete to the abstract. They might perform well with concrete materials and poorly with pencil and paper.

(3) A study should be made to determine what measurements should be taught in the school by surveying the uses made of measurement by the parents and community of the pupils. The need for changing from one unit of measure to another may be unnecessary for average adults to perform and the only reasons for measurement in the curriculum may be tradition.

(4) A study should be made to determine the methods of converting units of measure used by teachers of grades one through eight. Many teachers use their own unwritten methods for teaching a concept. One way to find out

what some of these are would be to interview a random sample of teachers.

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APPENDIX

MACHINE CALCULATION FORMULAS

Mean $\quad \bar{X} = \frac{\Sigma X}{N}$

Mean $\quad \bar{Y} = \frac{\Sigma Y}{N}$

Standard Deviation

$$s^2 = \frac{\Sigma X^2}{N} - \bar{X}^2$$

$$s = \sqrt{\frac{\Sigma X^2}{N} - \bar{X}^2}$$

Product - Moment Correlation r

$$r = \frac{[N\Sigma XY - \Sigma X \Sigma Y]}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{S_{\bar{X}}^2 + S_{\bar{Y}}^2 - 2 r_{xy} S_{\bar{X}} S_{\bar{Y}}}}$$

These formulas may be found in Smith, G. Milton. A Simplified Guide to Statistics. New York: Holt, Rinehart, and Winston, Inc., 1965.

NAME_____

CONVERSION TABLES

12 inches = 1 foot

3 feet = 1 yard

1 rod = 16.5 feet

5280 feet = 1 mile

5 cool = 1 wow

24 wow = 1 neat

6.5 whiz = 1 wow

4320 wow = 1 pow

ANSWERS

1. Change 6 rods to yards _____
2. Change 240 cool to neat _____
3. Change 4 pow to wow _____
4. Change 72 inches to yards _____
5. Change 4 miles to feet _____
6. Change 48 wow to neat _____
7. Change 6 pow to neat _____
8. Change 96 neat to wow _____
9. Change 4 wow to whiz _____
10. Change 9 feet to yards _____
11. Change 6 miles to yards _____
12. Change 15 yards to feet _____
13. Change 10 miles to inches _____
14. Change 10 cools to wow _____
15. Change 10 pow to cool _____
16. Change 24 inches to feet _____